SPECIALIZATION OF MORDELL-WEIL RANKS OVER SURFACES

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Abstract. Using the Shioda-Tate theorem and an adaptation of Silverman's specialization theorem, we reduce the specialization of Mordell-Weil ranks for abelian varieties over fields finitely generated over finitely generated fields to the the specialization theorem for Néron-Severi ranks recently proved by Ambrosi in positive characteristic. More precisely, we prove that after a blow-up of the base surface S, for all vertical curves S_x of a fibration $S \to U \subseteq \mathbf{P}^1_k$ with x from a subset of unbounded height of $|U|$, the Mordell-Weil rank of the Jacobian of a relative curve $\mathscr{C}/S$ over S stays the same when restricting to S_x.

1. Introduction

The Birch-Swinnerton-Dyer (BSD) conjecture relates in a surprising way properties of the L-function $L(A/K, s)$ of an abelian variety A over a finitely generated field K to arithmetic and geometric properties of A/K itself. In particular, it predicts that the vanishing order of $L(A/K, s)$ at $s = 1$ equals the rank of the finitely generated group $A(K)$. In the case where the characteristic p of K is positive, the BSD conjecture for A/K is equivalent to the finiteness of an ℓ-primary part of its Shafarevich-Tate group by work of many people, most recently [Qin20], where $\ell \neq p$. As the conjecture is more accessible when the (absolute or relative over a finitely generated field) transcendence degree of K is 1, it is desirable to investigate the behavior of BSD for A/K when K is specialized to a finitely generated field of lower transcendence degree.

In [Kel19, § 7] we proved using a Lefschetz hyperplane argument that the BSD conjecture for all abelian schemes $\mathscr{A}$ over all *surfaces* S over finite fields implies the BSD conjecture for all abelian schemes over all bases of dimension greater than two. However, the reduction to the case where the base is a *curve* could not be completed because we could not construct curves C on the surface S such that the ranks of $\mathscr{A}(S)$ and $\mathscr{A}(C)$ are equal. In the present article, we fill this gap.

In section 2 we merely observe that Silverman's specialization theorem [Sil83] holds in our setting, too, because we have the usual height machine from [Con06]. Section 3 is the core of this article; here, we prove that after blowing up the base surface S, there is a proper generically smooth fibration $S \to \mathbf{P}^1$ such that all but finitely many vertical curves S_x have the property that $\mathscr{A}(S) \otimes \mathbf{Q} \to \mathscr{A}(S_x) \otimes \mathbf{Q}$ is an isomorphism:

Theorem 1.1 (Theorem 3.8)**.** *Let k be a finitely generated field of positive transcendence degree over its prime field if it has positive characteristic. Let*

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$K|k$ be a finitely generated regular field extension and S/k a smooth separated (not necessarily proper) geometrically connected surface with function field K. Let A/K be an abelian variety with $\mathrm{Tr}_{K|k}(A) = 0$ or $\mathrm{Tr}_{K|k}(A)(k)$ finite, and $\mathscr{A}$ an extension of A to an abelian scheme over a dense open subscheme U of S.

Then for infinitely many divisors C on U, one has a specialization isomorphism

$$A(K) \otimes \mathbf{Q} \xrightarrow{\sim} \mathscr{A}(C) \otimes \mathbf{Q}$$

of rationalized Mordell-Weil groups.

(When $\mathscr{A}$ is the Jacobian of a relative curve $\mathscr{C}/S$, one can relax the conditions on $\mathscr{C}/S$: It is sufficient that it is a proper flat morphism with fibers geometrically connected curves and smooth generic fiber such that for all curves C on S, the restriction of $\mathscr{C}$ to C is a fibered surface as in Definition 2.1 below.)

We originally intended to use this theorem to verify the missing hypothesis in [Kel19, Theorem 7.0.3]. However, the reduction of the BSD conjecture for all abelian varieties over function fields of any transcendence degree over k to that of 1-dimensional function fields is already contained in [Gei19, Corollary 5.4].

Notation 1.2. The set of closed points of a scheme X is denoted by $|X|$. For a point v of a scheme, we denote its residue field by $\kappa(v)$.

2. The rank does not drop outside a set of bounded height

This section is only motivational because we deduce its main result Theorem 2.6 in the sharper form of Theorem 3.8 again in the next section.

We use the definition of a fibered surface from [Gor79, 2.1]:

Definition 2.1. A *fibered surface* $\mathscr{C} \to S$ over a field k is a smooth projective geometrically irreducible curve S/k, a proper smooth surface $\mathscr{C}/k$ and a proper flat morphism $\mathscr{C} \to S$ cohomologically flat in dimension 0 with fibers of dimension 1 and smooth projective geometrically irreducible generic fiber.

Remark 2.2. That $\pi : \mathscr{C} \to S$ is cohomologically flat in dimension 0 means that one has $\mathcal{O}_S \xrightarrow{\sim} \pi_*\mathcal{O}_{\mathscr{C}}$ universally. If the proper flat morphism π admits a section, one can omit the word 'universally'. See the remarks after the definition in [Gor79, 2.1].

For the definition of the $K|k$-trace see [Con06] and [Gor79, 4.2].

Theorem 2.3. *Let $\mathscr{C} \to S$ be a fibered surface with generic fiber C/K. Assume that the field extension $K|k$ is regular (primary in [Gor79, 4.2]). Then the $K|k$-trace of $A := \mathrm{Pic}^0_{C/K}$ is an abelian variety over k purely inseparably isogenous to $\mathrm{Pic}^0_{\mathscr{C}/k} / \mathrm{Pic}^0_{S/k}$.*

Proof. See [Gor79, Proposition 4.4]. □

Remark 2.4. The $K|k$-trace somewhat captures the constant part of $\mathscr{C}/X$, see [Con06, Example 2.2].

Theorem 2.5 (Mordell-Weil-Néron-Lang). *Let $K|k$ be a finitely generated regular field extension and A/K an abelian variety. Then $A(K)/\operatorname{Tr}_{K|k}(A)(k)$ is a finitely generated abelian group.*

Proof. See [Con06, Theorem 7.1]. □

As in [Waz06, text before Proposition 1], the height of a divisor on a smooth projective variety with respect to an embedding in projective space is its degree.

Theorem 2.6. *Let $K|k$ be a finitely generated regular field extension with smooth projective model S/k, A/K an abelian variety with $\operatorname{Tr}_{K|k}(A) = 0$ or $\operatorname{Tr}_{K|k}(A)(k)$ finite (e.g., k finite), and $\mathscr{A}$ an extension of A to an abelian scheme over a dense open subscheme U of S. For all $M > 0$, all but finitely many divisors $C \hookrightarrow U$ of degree $\leq M$, $A(K) \otimes \mathbf{Q} \to \mathscr{A}(C) \otimes \mathbf{Q}$ is injective.*

Proof. The theorem for k of characteristic 0 is [Waz06, Theorem 1 and the text before Proposition 1]. We merely describe the necessary changes when k is of positive characteristic: We only have to see that we have the 'height machine' for the arithmetic and geometric height in [Waz06] (which generalizes Silverman's specialization theorem [Sil83]).

The properties [Waz06, Proposition 2] of the 'arithmetic height machine' can be found for Conrad's generalized global fields in [Con06, text after Theorem 9.3] (note that (vi) is an immediate consequence of 'quasi-equivalence' since a curve has Néron-Severi group $\mathbf{Z}$). The 'canonical height machine' [Waz06, Proposition 3] for abelian varieties can be found in [Con06, text after Example 9.5].

The required properties of the 'geometric' height are proved in [Lan83, Chapter 6, Theorem 5.4] with the Northcott property in [Lan83, Chapter 3, Theorem 3.6] or [Con06, Lemma 10.3]. □

We will reprove this result in Theorem 3.8 for *vertical* divisors if S is realized as a fibered surface.

3. The rank does not grow outside a set of bounded height

In this section, S/k is a smooth, not necessarily proper, geometrically connected surface over a finitely generated field k of positive transcendence degree over its prime field if it has positive characteristic, and $\mathscr{C}/S$ a proper smooth morphism with fibers geometrically connected curves ("relative curve").

Using the following lemma, we can assume that there is a smooth fibration of our surface S/k to a non-empty open subscheme U of $\mathbf{P}^1_k$:

Lemma 3.1. *Let k be an infinite field and S/k a smooth separated geometrically connected surface (not necessarily proper). There is a blow-up $\tilde{S} \to \overline{S}$ with $S \hookrightarrow \overline{S}$ a proper compactification of S such that $\tilde{S}$ admits a proper flat morphism to $\mathbf{P}^1_k$ with smooth projective geometrically connected generic fiber.*

Proof. Since S is a smooth separated geometrically connected *surface*, it has a smooth projective compactification $S \hookrightarrow \overline{S}$ by Nagata compactification and resolution of singularities of surfaces, which can be achieved using successive blow-ups of smooth centers. Now the theory of Lefschetz pencils [SGA 7 II, Exposée XVII, Theorémè 2.5.2] (in characteristic 0, over finite fields, use [JS12, Theorem 2.2]) gives a blow-up of $\overline{S}$ together with a proper morphism to $\mathbf{P}^1_k$ with smooth generic fiber. $\square$

(A stronger version of Lemma 3.1 appears in [Waz06, Proposition 1], which holds for all infinite fields.)

Now restrict to a non-empty open subscheme U of $\mathbf{P}^1_k$ over which $\tilde{S} \to \mathbf{P}^1_k$ is smooth. The proper smooth relative curve $\mathscr{C} \to S$ can be extended to a proper smooth relative curve $\tilde{\mathscr{C}} = \mathscr{C} \times_S \tilde{S} \to \tilde{S}$ by functoriality of blow-ups [TS21, Lemma 085S]; this does not change the generic fiber. Remove from U the closed subscheme of points x such that $\mathscr{C} \times_U \{x\} =: \mathscr{C}_x \to S_x := S \times_U \{x\}$ is not smooth. This subset is not equal to U because $\mathscr{C} \to S \to U$ is generically smooth.

In the following, assume S/k is a smooth geometrically connected surface admitting a proper flat morphism to a non-empty open subscheme U of $\mathbf{P}^1_k$ with smooth and geometrically connected generic fibers. Consider the following situation, where all vertical arrows are smooth proper morphisms of relative dimension 1:

$$\begin{array}{ccccc}
\mathscr{C}_{k(U)} & \hookrightarrow & \mathscr{C} & \hookleftarrow & \mathscr{C}|_{S_x} \\
\downarrow & & \downarrow & & \downarrow \\
S_{k(U)} & \hookrightarrow & S & \hookleftarrow & S_x \\
\downarrow & & \downarrow & & \downarrow \\
\operatorname{Spec} k(U) & \hookrightarrow & U & \hookleftarrow & \{x\} \\
 & & \downarrow & & \\
 & & \operatorname{Spec} k & &
\end{array}$$

Here, $U \subseteq \mathbf{P}^1_k$ is a non-empty open subscheme such that $S|_U \to U$ is smooth; such an U exists because $S \to \mathbf{P}^1_k$ is generically smooth. We denote the restriction $S|_U$ again by S and the function field of U by $k(U)$. The right hand side of the diagram is constructed below; S_x is going to be the fiber of S/U over a closed point $x \in |U|$, and so a smooth projective vertical curve in S over U.

We denote the rank of the Néron-Severi group of a variety X by $\rho(X)$; it is finite by [SGA 6, Exp. XIII, § 5].

Lemma 3.2 (specialization of Néron-Severi rank)**.** *Assume k is a finitely generated field of transcendence degree ≥ 1 over $\mathbf{F}_p$ or of characteristic 0.*

There exists a $d \geq 1$ such that for infinitely many closed points x of U with $[\kappa(x) : k] \leq d$ (a complement of a sparse subset of U), the Néron-Severi rank of the special fiber $\mathscr{C}_x := \mathscr{C} \times_U \operatorname{Spec} \kappa(x)$ equals the Néron-Severi rank of the generic fiber $\mathscr{C}_{k(U)} := \mathscr{C} \times_U \operatorname{Spec} k(U)$ of the smooth proper relative surface $\mathscr{C} \to U$: $\rho(\mathscr{C}_x) = \rho(\mathscr{C}_{k(U)})$.

Proof. This follows from [Amb18, Corollary 1.7.1.3 (1)] (for k finitely generated of positive transcendence degree over $\mathbf{F}_p$) and [Cad13, Corollary 5.4] (for k of characteristic 0) applied to the smooth proper morphism $\mathscr{C} \to U$ with U a smooth and geometrically connected k-curve (a non-empty open subscheme of $\mathbf{P}^1_k$). $\square$

We now use the Shioda-Tate formula for the fibered surfaces $\mathscr{C}_x/\{x\}$ and $\mathscr{C}_{k(U)}/\operatorname{Spec} k(U)$ (note that these are indeed fibered surfaces!) to translate this equality of the Néron-Severi ranks of the generic and special fibers to an (in)equality of (between) the Mordell-Weil rank of the Jacobian of the relative curve $\mathscr{C}/S$ and the Mordell-Weil rank of $\mathscr{C}|_{S_x}/S_x$ with $S_x \hookrightarrow S$ the vertical smooth projective curve constructed as the closed fiber of $S \to U$ over $x \in |U|$.

Theorem 3.3 (Shioda-Tate formula)**.** *Let k be any field and $\mathscr{C} \to S$ a fibered surface over k. Call its generic fiber C/K, its Jacobian $\mathscr{A} := \operatorname{Pic}^0_{\mathscr{C}/X}$ and B/k the $K|k$-trace of $A = \operatorname{Jac}(\mathscr{C}_K)$. Then $\operatorname{rk}\operatorname{NS}(\mathscr{C}) = 2 + \operatorname{rk} A(K)/B(k) + \sum_v (h_v - 1)$ where h_v is the number of $k(v)$-rational components of its fiber.*

Proof. See [Gor79, Proposition 4.5 and its Corollary]. □

We first apply the Shioda-Tate formula Theorem 3.3 to the smooth proper surface $\mathscr{C}_x/\operatorname{Spec}\kappa(x)$ fibered over the curve S_x:

Lemma 3.4. *Let S_x be the smooth projective geometrically connected curve constructed as the closed fiber of $S \to U$ over $x \in |U|$.*

Then one has $\rho(\mathscr{C}_x) = 2 + \operatorname{rk}\mathscr{A}(S_x)/B(k) + \sum_{v\in|S_x|}(h_v - 1)$ with h_v the number of $\kappa(v)$-rational components of the fiber $\mathscr{C}_v$ of $\mathscr{C}_x \to S_x$ over $v \in |S_x|$.

If $\mathscr{C} \to S$ is a proper smooth relative curve, the error term $\sum_{v\in|S_x|}(h_v - 1)$ is 0.

Proof. The hypotheses of the Shioda-Tate formula Theorem 3.3 are satisfied for the surface $\mathscr{C}_x/\{x\}$ fibered over the curve S_x. If $\mathscr{C} \to S$ is a proper smooth relative curve, the error term vanishes trivially. □

We now apply the Shioda-Tate formula Theorem 3.3 to the smooth proper surface $\mathscr{C}_{k(U)}/\operatorname{Spec} k(U)$ fibered over the curve $S_{k(U)}$:

Lemma 3.5. *Denote the function field of S (equivalently, of $S_{k(U)}$) by K.*

One has $\rho(\mathscr{C}_{k(U)}) = 2 + \operatorname{rk}\mathscr{A}(K)/B(k)$.

Proof. The Shioda-Tate formula Theorem 3.3 applied to the fibered surface $\mathscr{C}_{k(U)}/\operatorname{Spec} k(U)$ fibered over the curve $S_{k(U)}/\operatorname{Spec} k(U)$ shows

$$\rho(\mathscr{C}_{k(U)}) = 2 + \operatorname{rk}\mathscr{A}(K)/B(k) + \sum_{v\in|S_{k(U)}|}(h_v - 1),$$

where h_v denotes the number of $\kappa(v)$-rational irreducible components of the fiber $\mathscr{C}_v$ of $\mathscr{C}_{k(U)}/S_{k(U)}$.

But a closed point $v \in S_{k(U)}$ of the generic fiber of S/U gives rise to a horizontal curve S_v/U by taking its closure in $S \supset S_{k(U)}$. Now if $\mathscr{C}_v = \mathscr{C}_{k(U)} \times_{S_{k(U)}} \{v\}$ were reducible, infinitely many of the closed fibers of the family $\mathscr{C}|_{S_v} \to S_v$ had reducible fibers, a contradiction to $\mathscr{C}/S$ being a fibered surface. Hence the error term vanishes. □

We now compare the Mordell-Weil group of the Jacobian $\mathscr{A}$ of the fibered curve $\mathscr{C}/S$ to the Mordell-Weil group of the Jacobian A of its generic fiber:

Lemma 3.6. *Restricting a section of the abelian scheme $\mathscr{A} \to S$ to the generic point of S gives an isomorphism $\mathscr{A}(S) \xrightarrow{\sim} A(K)$.*

Proof. Since S is regular and $\mathscr{A}/S$ is proper, by the valuative criterion for properness every element of $A(K)$ extends to a rational map $S \dashrightarrow \mathscr{A}$ defined outside a closed subset of codimension ≥ 2 in S, i.e., the locus of indeterminacy consists of a finite set of closed points. After a blow-up in a finite set of closed points of S, this becomes a morphism. But since S and the closed set is regular, the exceptional divisors are projective spaces, which admit only constant morphisms to abelian varieties. Hence $S \dashrightarrow \mathscr{A}$ extends uniquely to a section of $\mathscr{A}/S$, i.e., one has a homomorphism $A(K) \to \mathscr{A}(S)$ inverse to the restriction $\mathscr{A}(S) \to A(K)$. □

We now combine the previous results to the main result of this note, an equality between the Mordell-Weil ranks of $\mathscr{A}/S$ and $\mathscr{A}|_{S_x}/S_x$:

Corollary 3.7. *Assume that the $K|k$-trace B is trivial. After blowing up S and possibly shrinking it, such that there is a morphism $S \to U$ as in Lemma 3.1, there exists a $d \geq 1$ such that for infinitely many closed points x of U with $[\kappa(x) : k] \leq d$ (a complement of a sparse subset of U), one has* $\operatorname{rk} A(K) \geq \operatorname{rk} \mathscr{A}(S_x)$.

If $\mathscr{C} \to S$ is a proper smooth relative curve, one has equality $\operatorname{rk} A(K) = \operatorname{rk} \mathscr{A}(S_x)$.

Proof. One has

$$\operatorname{rk} \mathscr{A}(S) = \operatorname{rk} A(K) \geq \operatorname{rk} \mathscr{A}(S_x),$$

where the equality holds by Lemma 3.6 and the inequality is a combination of by Lemmata 3.2, 3.4 and 3.5 for all $x \in |U|$ except from a set of bounded height. □

(Replacing Lemma 3.1 by [Waz06, Proposition 1], one gets this corollary for *all* divisors.)

Since every abelian variety over an infinite field is a quotient of a Jacobian, this easily generalizes to:

Theorem 3.8. *Let k be a finitely generated field of positive transcendence degree over its prime field if it has positive characteristic. Let $K|k$ be a finitely generated regular field extension and S/k a smooth separated (not necessarily proper) geometrically connected surface with function field K. Let A/K be an abelian variety with* $\operatorname{Tr}_{K|k}(A) = 0$ *or* $\operatorname{Tr}_{K|k}(A)(k)$ *finite, and $\mathscr{A}$ an extension of A to an abelian scheme over a dense open subscheme U of S.*

Then for infinitely many divisors C on U, one has a specialization isomorphism

$$A(K) \otimes \mathbf{Q} \xrightarrow{\sim} \mathscr{A}(C) \otimes \mathbf{Q}$$

of rationalized Mordell-Weil groups.

Proof. By [Mil86, Theorem 10.1] (note that K is infinite), there exists a smooth projective geometrically connected curve C over K and a surjective homomorphism $\operatorname{Pic}^0_{C/K} \twoheadrightarrow A$. Since the isogeny category of abelian varieties is semisimple, $\operatorname{Pic}^0_{C/K}$ is isogenous to a product $A \times_K B$ of abelian varieties.

We use that the intersection of the set of vertical divisors S_x in Corollary 3.7 and the divisors in Theorem 2.6 is infinite: For $x \in |U|$ with degree $[\kappa(x) : k]$ bounded, infinitely many of the S_x satisfy the statement in Corollary 3.7, so by the Northcott property of the height used in Theorem 2.6, the height of these S_x is unbounded, so there must be infinitely many of them.

In the following use that the Mordell-Weil rank does not change under isogenies. By spreading out and possibly shrinking S, one obtains an isogeny $\mathrm{Pic}^0_{\mathscr{C}/S} \to \mathscr{A} \times_S \mathscr{B}$. By Corollary 3.7 for $\mathscr{C}/S$ and the S_x there and because the rank is additive,

$$\mathrm{rk}\,\mathscr{A}(S) + \mathrm{rk}\,\mathscr{B}(S) = \mathrm{rk}\,\mathrm{Pic}^0_{\mathscr{C}/S}(S) = \mathrm{rk}\,\mathrm{Pic}^0_{\mathscr{C}/S}(S_x) = \mathrm{rk}\,\mathscr{A}(S_x) + \mathrm{rk}\,\mathscr{B}(S_x).$$

But by Theorem 2.6, $\mathrm{rk}\,\mathscr{A}(S) \leq \mathrm{rk}\,\mathscr{A}(S_x)$ for x from a set of unbounded height and analogously for $\mathscr{B}$, so by the previous equation, one must have equality for both. The injectivity of the rationalized specialization morphisms together with the equality of ranks implies that they are isomorphisms. □

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References

[Amb18] Emiliano Ambrosi, *Specialization of Néron-Severi groups in positive characteristic*, 2018. Preprint, arXiv:1810.06481. ↑4

[Cad13] Anna Cadoret, *Motivated cycles under specialization*, Geometric and differential Galois theories, Sémin. Congr., vol. 27, Soc. Math. France, Paris, 2013, pp. 25–55 (English, with English and French summaries). MR3203548 ↑4

[Con06] Brian Conrad, *Chow's K/k-image and K/k-trace, and the Lang-Néron theorem*, Enseign. Math. (2) **52** (2006), no. 1-2, 37–108. MR2255529 ↑1, 2, 3

[Gei19] Thomas H. Geisser, *Tate's conjecture and the Tate-Shafarevich group over global function fields*, Journal of the Institute of Mathematics of Jussieu, posted on 2019, 1–22, DOI 10.1017/S147474801900046X. ↑2

[Gor79] W. J. Gordon, *Linking the conjectures of Artin-Tate and Birch-Swinnerton-Dyer*, Compositio Math. **38** (1979), no. 2, 163–199. MR528839 ↑2, 5

[JS12] Uwe Jannsen and Shuji Saito, *Bertini theorems and Lefschetz pencils over discrete valuation rings, with applications to higher class field theory*, J. Algebraic Geom. **21** (2012), no. 4, 683–705, DOI 10.1090/S1056-3911-2012-00570-0. MR2957692 ↑4

[Kel19] Timo Keller, *On an analogue of the conjecture of Birch and Swinnerton-Dyer for Abelian schemes over higher dimensional bases over finite fields*, Doc. Math. **24** (2019), 915–993. MR3992320 ↑1, 2

[Lan83] Serge Lang, *Fundamentals of Diophantine geometry*, Springer-Verlag, New York, 1983. MR715605 ↑3

[Mil86] J. S. Milne, *Jacobian varieties*, Arithmetic geometry (Storrs, Conn., 1984), Springer, New York, 1986, pp. 167–212. MR861976 ↑6

[Qin20] Yanshuai Qin, *Comparison of different Tate conjectures*, 2020. Preprint, arXiv:2012.01337. ↑1

[Sil83] Joseph H. Silverman, *Heights and the specialization map for families of abelian varieties*, J. Reine Angew. Math. **342** (1983), 197–211, DOI 10.1515/crll.1983.342.197. MR703488 ↑1, 3

[Waz06] R. Wazir, *On the specialization theorem for abelian varieties*, Bull. London Math. Soc. **38** (2006), no. 4, 555–560, DOI 10.1112/S0024609306018261. MR2250746 ↑3, 4

[TS21] The Stacks project authors, *The Stacks project*, 2021. https://stacks.math.columbia.edu. ↑4

[SGA 6] Pierre Berthelot, Alexandre Grothendieck, and Luc Illusie, *Séminaire de Géométrie Algébrique du Bois Marie – 1966–67 – Théorie des intersections et théorème de Riemann-Roch – (SGA 6)*, Lecture notes in mathematics (in French) **225**, Berlin; New York: Springer-Verlag. xii+700, 1971 (french). ↑4

[SGA 7 II] *Groupes de monodromie en géométrie algébrique. II*, Lecture Notes in Mathematics, Vol. 340, Springer-Verlag, Berlin-New York, 1973 (French). Séminaire de Géométrie Algébrique du Bois-Marie 1967–1969 (SGA 7 II); Dirigé par P. Deligne et N. Katz. MR0354657 ↑3

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