Kleine AG: Tannakian categories

Organisation:
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The starting point for the theory of Tannakian categories is the Tannaka reconstruction theorem. It says that any affine group scheme G over a field k can be reconstructed from the group of automorphisms of the forgetful functor $\omega : \mathrm{Rep}_k(G) \to \mathrm{Vect}_k$ which commute with tensor products. Tannakian categories are a formalisation of this "reconstruction data". Its theory has many applications in algebraic geometry and number theory.

1. Tannaka reconstuction theorem (45 minutes)

The aim of this talk is to give a detailed proof of the Tannaka reconstruction theorem (e.g. [DM] Prop. 2.8) in down to earth terms *without using the notion of a tensor category*. Before you prove the theorem, recall the theory of representations of affine group schemes as presented in [DM] sect. 2.

2. Tensor categories (45 minutes)

Present tensor categories as in [DM] sect. 1. Make sure to give some examples and to include the subsections about abelian tensor categories, rigid tensor categories, tensor functors and morphisms of tensor functors. Also reformulate the Tannaka reconstruction theorem in the language of tensor categories.

3. Neutral Tannakian categories (45 minutes)

Give the definition of a neutral Tannakian category (cf. [DM] Def. 2.19) and prove [DM] Thm. 2.11.

4. Tannakian categories (45 minutes)

Give the definition of a fiber functor and state (*not* prove) [DM] Thm. 3.2. Give the definition of a Tannakian category ([DM] Def. 3.8) and explain all notions needed for this definition. Also give the example of a neutral Tannakian category as Tannakian category.

5. Application (30-60 minutes)

Motivate and give **one** application of Tannakian categories. Possible applications include:

- The definition of the Newton point ([Kot]) motivated by crystalline cohomology.
- Show that any differential equation of order n in a differential field (K, ∂) with an algebraically closed field of constants has n linear independent solutions after a finite field extension as in [Del] ch. 9.
- Any application you know in your own area of expertise.

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References

[Del] P. Deligne: Catégories Tannakiennes, The Grothendieck Festschrift, Vol. II, 111-195, Progr. Math., 87, Birkhäuser Boston, Boston, MA, 1990.

[DM] P. Deligne and J. S. Milne: Tannakian categories, `http://www.jmilne.org/math/xnotes/tc.pdf`

[Kot] R. Kottwitz: Isocrystals with additional structure, Compositio Math. **56** (1985), no. 2, 201-220