BAYERISCHE KLEINE AG: STACKS

Organization:
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The goal of this *kleine AG* is to give an introduction to stacks.

First talk. Motivation.

- Explain why there is no scheme representing the fine moduli space of elliptic curves. ([HaMo], 2.A and [EiHa], VI.2.4)
- Discuss possible solutions (level-n structures M_n, [DeRa], I)
- Explain why one needs a suitable definition of a quotient of M_n by $GL_2(\mathbf{Z}/n)$.

Second talk. Grothendieck topologies.

- Explain [Vi], 2.3.1–2.3.3 (2.3.2 only very rough, it suffices to cite 2.33(i), (ii) and 2.35, 2.36)
- Representable functors are fpqc sheaves 2.55
- Characterisation of representable functors [EiHa], Theorem VI-14.

Third talk. Fibered Categories.

- Introduce the notion of a category fibered in groupoids. See [DeMu, 4], for a more detailed exposition look at Sections 3.1.1 and 3.3 in [Vi].
- Interpret a category fibered in groupoids (CFG) in terms of pseudo-functors / lax 2-functors. Conversely, associate a CFG to a groupoid-valued pseudo-functor. See [Vi, 3.1.2, 3.1.3].
- Give a few examples of CFGs, e.g. vector bundles, quotients by group actions, classifying stacks (cf. [Vi, 3.2] and [Gó, 2.1, 2.2]).

Fourth talk. Definition of Stacks.

- Discuss the category of descent data of CFGs and give the definition of stacks, as explained in [Vi, 4.1.2, 4.1.3]. If we were dealing with categories fibered in sets, a stack would be nothing but a sheaf, see [Vi, 3.4 and Prop. 4.9].
- Representable stacks and the 2-Yoneda lemma, [Vi, 3.6.1, 3.6.2]. The category C is a full sub-2-category of the category of stacks over C.
- Follow [DeMu, 4]: „S-Stacks“, representable 1-morphisms of stacks, quasi-separated stacks, finally the definition of algebraic stacks. Note that in today's terminology this would be a Deligne-Mumford stack. Algebraic stacks usually refer to Artin stacks. Briefly compare the definitions of Deligne-Mumford stacks and Artin stacks as in [Gó, 2.3].

Fifth talk. Properties of Stacks. The reference here is [DeMu, 4].

- Explain how to transfer geometrical notions from the realm of schemes to algebraic stacks, in particular „separated“, „quasi-compact“, „of finite type“, „proper“.
- Define topological concepts for algebraic stacks: open and closed substacks, connected and irreducible components.
- Cite the valuative criterion for separatedness and properness.

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Sixth talk. Stable curves as a Deligne-Mumford stack. As an application, we show that the stack of stable curves of genus g is a Deligne-Mumford stack. Our main source is [Ed], but confer also [DeMu].

- Theorem 2.1 in [Ed]
- Corollary 2.2 in [Ed]
- Section 3.1 in [Ed]
- Explain shortly why M_g is proper, [Ed] section 3.2.

References

[DeMu] P. Deligne, D. Mumford, *The irreducibility of the space of curves of given genus*, Publ. Math. IHES 36, 75-110 (1969)

[DeRa] P. Deligne, M. Rapoport, *Les schmas de modules de courbes elliptiques*, in: Modular Functions of One Variable II, Springer LNM 349, Berlin, Springer-Verlag (1973)

[Ed] D. Edidin, *Notes on the construction of the moduli spaces of curves*, `arXiv:math/9805101`

[EiHa] D. Eisenbud, J. Harris, *The Geometry of Schemes*, Springer GTM 197, New-York, Springer-Verlag (2000)

[Gó] T. L. Gómez, *Algebraic Stacks*, `arXiv:math/9911199v1`

[HaMo] J. Harris, I. Morrison, *Moduli of Curves*, Springer GTM 187, New-York, Springer-Verlag (1998)

[Vi] A. Vistoli, *Grothendieck topologies, fibered categories and descent theory*, in: Fundamental Algebraic Geometry – Grothendieck's FGA Explained, Providence, RI, AMS (2005)