KLEINE AG: CRYSTALLINE COHOMOLOGY

Organization:
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The goal of this *kleine AG* is to give an introduction to crystalline cohomology as developed in the first chapters of [BO]. The last talk will give an overview of results and applications.

First talk. Differential operators, connections and stratifications. (60 minutes)
This talk presents the classical concepts of differential operators, connections and stratifications following chapter 2 of [BO]. Special emphasis should be put on proposition 2.11 which establishes the equivalence of stratifications with sheaves on the infinitesimal site and morphisms of differential operators. Another important result is theorem 2.15 which examines whether a connection can be extended to a stratification. These two are the results we want to mimic in characteristic p in the following talks using divided power algebras.

Second talk. Divided powers. (60 minutes)
This talk introduces the concept of divided power algebras, also called P.D.-algebras as presented in chapter 3 of [BO]. The goal is to construct for a P.D.-algebra A and an ordinary A-algebra B the P.D-envelope of B over A which can be thought of as the P.D.-algebra generated by B. This construction is done in theorem 3.19. You should also discuss nilpotent P.D. ideals as presented in the last part of chapter 3.

Third talk. P.D.-Stratifications. (30 minutes)
This talk follows chapter 4 of [BO]. It introduces P.D.-stratifications and treats the P.D.-analogue of proposition 2.11, namely theorem 4.8. The important fact is that in the P.D.-setting any integrable connection extends to a P.D.-stratification without divisibility assumptions on the base scheme.

Fourth talk. Definition of crystalline cohomology. (60 minutes)
Following chapter 5 of [BO], this talk defines the crystalline topos and thus crystalline cohomology.

Fifth talk. Properties of crystalline cohomology. (60 minutes)
Prove for motivation that for $\ell \neq \operatorname{char} k$, the Kummer sequence over k separably closed

$$1 \to \mu_{\ell^n} \to \mathbf{G}_m \to \mathbf{G}_m \to 1$$

yields

$$H^1_{et}(X, \mathbf{Z}_\ell(1)) = T_\ell \mathrm{Pic}(X).$$

Now we want to prove an analogous result for crystalline cohomology. Sketch as much as possible from the proof of [Il], p. 618 f. Remarque 3.11.2: If A is an Abelian variety, there is an isomorphism $H^1(A/W)$ to the Dieudonné module of the p-divisible group of A.

References

[BO] Berthelot P., Ogus A. *Notes on cristalline cohomology*, 1978.
[Il] Illusie L., *Complexe de de Rham-Witt et cohomologie cristalline*, 1979. `http://archive.numdam.org/ARCHIVE/ASENS/ASENS_1979_4_12_4/ASENS_1979_4_12_4_501_0/ASENS_1979_4_12_4_501_0.pdf`

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