KLEINE AG: FINITENESS THEOREMS FOR BRAUER GROUPS OF K3 SURFACES

Organization:
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The goal of this *Bayerische Kleine AG* is to give a thorough proof of the following Theorem 1.2 in [SZ]: Let k be a finitely generated field over $\mathbf{Q}$ and $\pi : X \to k$ a K3 surface. Then $\mathrm{Br}(X)/\pi^*(\mathrm{Br}(k))$ is finite.

It is enough to show that $\mathrm{Br}(\overline{X})^\Gamma$ is finite (where $\Gamma = \mathrm{Gal}(\overline{k}/k)$)

The proof of this can be divided into three parts:

(1) $\mathrm{H}^1(k, \mathrm{NS}(\overline{X}) \otimes \mathbb{Z}/\ell) \to \mathrm{H}^1(k, \mathrm{H}^2_{\text{ét}}(\overline{X}, \mu_\ell))$ is injective for almost all primes ℓ.

(2) $\mathrm{Br}(\overline{X})^\Gamma(\ell)$ is finite for all primes ℓ

(3) $(\mathrm{NS}(\overline{X})/\ell)^\Gamma \to \mathrm{H}^2_{\text{ét}}(\overline{X}, \mu_\ell)$ is an isomorphism for almost all primes ℓ.

Part (1) and (3) imply that $\mathrm{Br}(\overline{X})^\Gamma(\ell) = 0$ for almost all primes ℓ. Together with (2), the theorem follows.

First talk. Basic facts about Brauer groups. (60 minutes)

Introduce and give some background information on the (cohomological) Brauer group of an algebraic variety (see for example [Mi]). Explain why, for the proof of the theorem, it is enough to show that $\mathrm{Br}(\overline{X})^\Gamma$ is finite (where $\Gamma = \mathrm{Gal}(\overline{k}/k)$) (Remark 1.3). Introduce the sequences in section 2.2 (In particular the important sequence (5))

Second talk. The first step. (60 minutes)

The goal of this talk is to prove part (1) as described in the introduction and prove part (2) in the case of Abelian varieties.

Looking at [SZ], this means the following: Prove Corollaries 2.6 and 2.7 in [SZ] by using/proving the required Lemmas and Corollaries in section 2 in the case of characteristic 0. This includes in particular the first part of Lemma 2.3 and Proposition 2.5 a.

Third talk. The second step. (45 minutes)

Prove part (2) of the introduction in the case of K3 surfaces in characteristic 0 and prepare the proof of step (3).

Again, in terms of [SZ]: Give a rough idea why Lemma 4.4 follows from Corollary 2.7 (i.e. fill in some details in the proof of Lemma 4.4). Prepare the proof of Lemma 4.3 by explaining the necessary comparison theorems in [SZ] on p. 494-496.

Fourth talk. The third step. (60 minutes)

All that remains is step (3).

In terms of [SZ]: Prove Lemma 4.3 using the previous talks. Explain again why Theorem 1.2 follows.

References

[SZ] Skorobogatov A., Zarhin, Y. *A finiteness theorem for the Brauer group of Abelian varieties and K3 surfaces*, J. Alg. Geom., **17**, 2008. http://arxiv.org/abs/math/0605351

[Mi] J.S.Milne *Étale cohomology, Chapter IV*, Princeton Univ. Press, Princeton, N.J., 1980.

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